

Indian Statistical Institute, Bangalore

B. Math (Hons.) Third Year

First Semester - Combinatorics and graph theory

Midterm Exam

Date: September 14, 2018

Maximum marks: 100

Duration: 3 hours

Instruction: Answer any five questions.

1. For each of the following parameters v, κ, λ (given in this order) decide (with full proof) whether a 2-design exists or not:- [4 x 5 = 20]
 - (a) 6, 3, 3
 - (b) 6, 3, 2
 - (c) 11, 5, 2
 - (d) 21, 6, 1
 - (e) 22, 7, 2
2. (a) Prove that every $2 - (n^2, n, 1)$ design ($n \geq 2$) satisfies Playfair's axiom.
(b) Prove that every finite affine plane is the residual of a finite projective plane. [10 + 10 = 20]
3. (a) Let $\lambda \geq 2$. Let D be a finite incidence system satisfying
 - i. any two distinct points of D are together incident with exactly λ blocks, and
 - ii. any two distinct blocks D are together incident with exactly λ points.Then show that D is a square 2-design provided D has more than λ points and more than λ blocks.
(b) Give an infinite family of examples to show that the result of part (a) is false for $\lambda = 1$. [10 + 10 = 20]
4. Let D be a 2-design with $v = \binom{k+1}{2}$ and $\lambda = 2$. Then show that any two blocks of D have one or two common points. [20]
5. Let $q \equiv 3 \pmod{4}$ be a prime power. Let S be the set of all non-zero squares in the field of order q . Then show that the additive translates of S are the blocks of a square 2-design. [20]
6. Use Fisher equations to prove Fisher's inequality. [20]